

MATH 141: Quiz 6

Name: key

Directions:

- * Show your thought process (commonly said as "show your work") when solving each problem for full credit. **Remember to fully simplify.**
- * If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- * Good luck!

1. Suppose

$$f(x) = x - \frac{1}{x}$$

Find the intervals of concavity.

① Find potential inflection points.

$$f(x) = x - \frac{1}{x} = x - x^{-1}$$

$$f'(x) = 1 + x^{-2}$$

$$f''(x) = 0 - 2x^{-3} = -\frac{2}{x^3}$$

Ⓐ solve $f''(x) = 0$

$$x^3 \cdot -\frac{2}{x^3} = 0 \cdot x^3$$

$$-2 = 0$$

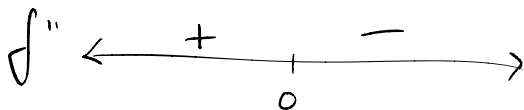
No solution

Ⓑ $f''(x)$ DNE. Set denom = 0 and solve.

$$(x^3)^{\frac{1}{3}} = (0)^{\frac{1}{3}}$$

$$x = \sqrt[3]{0} = 0$$

② Sign diagram of $f''(x)$



$$f''(-1) = -\frac{2}{(-1)^3} = -\frac{+}{-} = +$$

$$f''(1) = -\frac{2}{1^3} = -$$

$\therefore f(x)$ is concave up on $(-\infty, 0)$
concave down on $(0, \infty)$

2. Given the function

$$f(x) = x^4 - 6x^2$$

- (a) Find the intervals on which $f(x)$ are increasing and decreasing.
 (b) Find all relative extrema (not just the x-coordinate).
 (c) Find the intervals of concavity.
 (d) Find the inflection points.

(a) + (b): ① Crit #'s

$$f'(x) = 4x^3 - 12x = 4x(x^2 - 3)$$

② Solve $f'(x) = 0$

$$4x(x^2 - 3) = 0$$

$$4x = 0$$

$$x = 0$$

$$x^2 - 3 = 0$$

$$\sqrt{x^2} = \pm\sqrt{3}$$

$$x = \pm\sqrt{3}$$

③ $f'(x)$ DNE

N/A

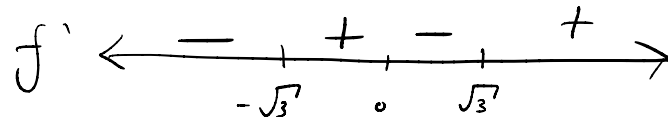
② Sign diagram of $f'(x)$

$$f'(-2) = 4(-2)((-2)^2 - 3) = - \cdot + = -$$

$$f'(-1) = 4(-1)((-1)^2 - 3) = - \cdot - = +$$

$$f'(1) = 4 \cdot 1(1^2 - 3) = + \cdot - = -$$

$$f'(2) = 4 \cdot 2(2^2 - 3) = +$$



$\therefore f(x)$ decreasing on $(-\infty, -\sqrt{3}) \cup (0, \sqrt{3})$
 increasing on $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$

Local minima of

$$f(-\sqrt{3}) = (-\sqrt{3})^4 - 6(-\sqrt{3})^2 = 9 - 6 \cdot 3 = -9$$

$$f(\sqrt{3}) = (\sqrt{3})^4 - 6(\sqrt{3})^2 = -9$$

Local maximum of

$$f(0) = 0^4 - 6 \cdot 0^2 = 0$$

(c) + (d) ① Find potential inflection points

$$f''(x) = 12x^2 - 12 = 12(x^2 - 1) = 12(x-1)(x+1)$$

② Solve $f''(x) = 0$

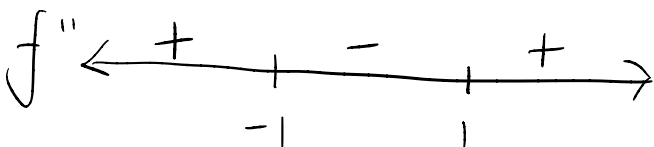
$$12(x-1)(x+1) = 0$$

$$x = -1, 1$$

③ $f''(x)$ DNE

N/A, domain \mathbb{R}

② Sign diagram of $f''(x)$



$$f''(-2) = 12(-2-1)(-2+1) = +$$

$$f''(0) = 12 \cdot (0-1)(0+1) = -$$

$$f''(2) = 12 \cdot (2-1)(2+1) = +$$

$\therefore f(x)$ concave down on $(-1, 1)$
 concave up on $(-\infty, -1) \cup (1, \infty)$

Inflection points

$$(1, f(1)) = (1, -5)$$

$$(-1, f(-1)) = (-1, -5)$$